

## 2014 Maths Paper1 Memo

Let  $\rho$  be the automorphic representation of  $GL_2$  generated by a full level cuspidal Siegel eigenform that is not a Saito-Kurokawa lift, and  $\sigma$  be an arbitrary cuspidal, automorphic representation of  $GL_2$ . Using Furusawa's integral representation for  $L$  combined with a pullback formula involving the unitary group  $U_2$ , the authors prove that the  $L$ -functions are "nice". The converse theorem of Cogdell and Piatetski-Shapiro then implies that such representations have a functorial lifting to a cuspidal representation of  $GL_2$ . Combined with the exterior-square lifting of Kim, this also leads to a functorial lifting of  $\rho$  to a cuspidal representation of  $GL_2$ . As an application, the authors obtain analytic properties of various  $L$ -functions related to full level Siegel cusp forms. They also obtain special value results for  $L$  and  $\sigma$ .

"The text is suitable for a typical introductory algebra course, and was developed to be used flexibly. While the breadth of topics may go beyond what an instructor would cover, the modular approach and the richness of content ensures that the book meets the needs of a variety of programs."--Page 1.

Joseph and Hodges-Levasseur (in the  $A$  case) described the spectra of all quantum function algebras on simple algebraic groups in terms of the centers of certain localizations of quotients of  $GL_n$  by torus invariant prime ideals, or equivalently in terms of orbits of finite groups. These centers were only known up to finite extensions. The author determines the centers explicitly under the general conditions that the deformation parameter is not a root of unity and without any restriction on the characteristic of the ground field. From it he deduces a more explicit description of all prime ideals of  $GL_n$  than the previously known ones and an explicit parametrization of  $GL_n$ .

There exist results on the connection between the theory of wavelets and the theory of integral self-affine tiles and in particular, on the construction of wavelet bases using integral self-affine tiles. However, there are many non-integral self-affine tiles which can also yield wavelet basis. In this work, the author gives a complete characterization of all one and two dimensional  $\lambda$ -dilation scaling sets such that  $\lambda$  is a self-affine tile satisfying for some  $R$ , where  $A$  is a integral expansive matrix with  $\det A \neq 0$  and  $\lambda \in \mathbb{R}$ .

The author studies a family of renormalization transformations of generalized diamond hierarchical Potts models through complex dynamical systems. He proves that the Julia set (unstable set) of a renormalization transformation, when it is treated as a complex dynamical system, is the set of complex singularities of the free energy in statistical mechanics. He gives a sufficient and necessary condition for the Julia sets to be disconnected. Furthermore, he proves that all Fatou components (components of the stable sets) of this family of renormalization transformations are Jordan domains with at most one exception which is completely invariant. In view of the problem in physics about the distribution of these complex singularities, the author proves here a new type of distribution: the set of these complex singularities in the real temperature domain could contain an interval. Finally, the author studies the boundary behavior of the first derivative and second derivative of the free energy on the Fatou component containing the infinity. He also gives an explicit value of the second order critical exponent of the free energy for almost every boundary point.

This paper quantifies the speed of convergence and higher-order asymptotics of fast diffusion dynamics on  $\mathbb{R}^n$  to the Barenblatt (self similar) solution. Degeneracies in the parabolicity of this equation are cured by re-expressing the dynamics on a manifold with a cylindrical end, called the cigar. The nonlinear evolution becomes differentiable in Hölder spaces on the cigar. The linearization of the dynamics is given by the Laplace-Beltrami operator plus a transport term (which can be suppressed by introducing appropriate weights into the function space norm), plus a finite-depth potential well with a universal profile. In the limiting case of the (linear) heat equation, the depth diverges, the number of eigenstates increases without bound, and the continuous spectrum recedes to infinity. The authors provide a detailed study of the linear and nonlinear problems in Hölder spaces on the cigar, including a sharp boundedness estimate for the semigroup, and use this as a tool to obtain sharp convergence results toward the Barenblatt solution, and higher order asymptotics. In finer convergence results (after modding out symmetries of the problem), a subtle interplay between convergence rates and tail behavior is revealed. The difficulties involved in choosing the right functional spaces in which to carry out the analysis can be interpreted as genuine features of the equation rather than mere annoying technicalities.

The authors study the following singularly perturbed problem: in  $\mathbb{R}^n$ . Their main result is the existence of a family of solutions with peaks that cluster near a local maximum of  $f$ . A local variational and deformation argument in an infinite dimensional space is developed to establish the existence of such a family for a general class of nonlinearities  $F$ .

The authors study the perturbation of a shock wave in conservation laws with physical viscosity. They obtain the detailed pointwise estimates of the solutions. In particular, they show that the solution converges to a translated shock profile. The strength of the perturbation and that of the shock are assumed to be small but independent. The authors' assumptions on the viscosity matrix are general so that their results apply to the Navier-Stokes equations for the compressible fluid and the full system of magnetohydrodynamics, including the cases of multiple eigenvalues in the transversal fields, as long as the shock is classical. The authors' analysis depends on accurate construction of an approximate Green's function. The form of the ansatz for the perturbation is carefully constructed and is sufficiently tight so that the author can close the nonlinear term through Duhamel's principle.

The little  $n$ -disks operad,  $\mathcal{D}_n$ , along with its variants, is an important tool in homotopy theory. It is defined in terms of configurations of disjoint  $n$ -dimensional disks inside the standard unit disk in  $\mathbb{C}$  and it was initially conceived for detecting and understanding  $n$ -fold loop spaces. Its many uses now stretch across a variety of disciplines including topology, algebra, and mathematical physics. In this paper, the authors develop the details of Kontsevich's proof of the formality of little  $n$ -disks operad over the field of real numbers. More precisely, one can consider the singular chains on  $\mathcal{D}_n$  as well as the singular homology of  $\mathcal{D}_n$ . These two objects are operads in the category of chain complexes. The formality then states that there is a zig-zag of quasi-isomorphisms connecting these two operads. The formality also in some sense holds in the category of commutative differential graded algebras. The authors additionally prove a relative version of the formality for the inclusion of the little  $n$ -disks operad in the little  $m$ -disks operad when  $n < m$ .

The authors define combinatorial Floer homology of a transverse pair of noncontractible nonisotopic embedded loops in an oriented  $3$ -manifold without boundary, prove that it is invariant under isotopy, and prove that it is isomorphic to the original Lagrangian Floer homology. Their proof uses a formula for the Viterbo-Maslov index for a smooth lune in a  $3$ -manifold.

Hua's fundamental theorem of geometry of matrices describes the general form of bijective maps on the space of all  $m \times n$  matrices over a division ring  $\mathbb{D}$  which preserve adjacency in both directions. Motivated by several applications the author studies a long standing open problem of possible improvements. There are three natural questions. Can we replace the assumption of preserving adjacency in both directions by the weaker assumption of preserving adjacency in one direction only and still get the same conclusion? Can we relax the bijectivity assumption? Can we obtain an analogous result for maps acting between the spaces of rectangular matrices of different sizes? A division ring is said to be EAS if it is not isomorphic to any proper subring. For matrices over EAS division rings the author solves all three problems simultaneously, thus obtaining the optimal version of Hua's theorem. In the case of general division rings he gets such an optimal result only for square matrices and gives examples showing that it cannot be extended to the non-square case.

In this paper the authors extend the notion of a continuous bundle random dynamical system to the setting where the action of  $\mathbb{R}$  or  $\mathbb{N}$  is replaced by the action of an infinite countable discrete amenable group. Given such a system, and a monotone sub-additive invariant family of random continuous functions, they introduce the concept of local fiber topological pressure and establish an associated variational principle, relating it to measure-theoretic entropy. They also discuss some variants of this variational principle. The authors introduce both

topological and measure-theoretic entropy tuples for continuous bundle random dynamical systems, and apply variational principles to obtain a relationship between these of entropy tuples. Finally, they give applications of these results to general topological dynamical systems, recovering and extending many recent results in local entropy theory.

This book, *Teaching Learners with Visual Impairment*, focuses on holistic support to learners with visual impairment in and beyond the classroom and school context. Special attention is given to classroom practice, learning support, curriculum differentiation and assessment practices, to mention but a few areas of focus covered in the book. In this manner, this book makes a significant contribution to the existing body of knowledge on the implementation of inclusive education policy with learners affected by visual impairment.

Let  $F$  be a non-Archimedean local field. Let  $\mathcal{W}_F$  be the Weil group of  $F$  and  $\mathcal{P}_F$  the wild inertia subgroup of  $\mathcal{W}_F$ . Let  $\widehat{\mathcal{W}}_F$  be the set of equivalence classes of irreducible smooth representations of  $\mathcal{W}_F$ . Let  $\mathcal{A}^0_n(F)$  denote the set of equivalence classes of irreducible cuspidal representations of  $\mathrm{GL}_n(F)$  and set  $\widehat{\mathcal{GL}}_n = \bigcup_{n \geq 1} \mathcal{A}^0_n(F)$ . If  $\sigma \in \widehat{\mathcal{W}}_F$ , let  $^L\sigma \in \widehat{\mathcal{GL}}_n$  be the cuspidal representation matched with  $\sigma$  by the Langlands Correspondence. If  $\sigma$  is totally wildly ramified, in that its restriction to  $\mathcal{P}_F$  is irreducible, the authors treat  $^L\sigma$  as known. From that starting point, the authors construct an explicit bijection  $\mathbb{N} : \widehat{\mathcal{W}}_F \rightarrow \widehat{\mathcal{GL}}_n$ , sending  $\sigma$  to  $^N\sigma$ . The authors compare this "naïve correspondence" with the Langlands correspondence and so achieve an effective description of the latter, modulo the totally wildly ramified case. A key tool is a novel operation of "internal twisting" of a suitable representation  $\pi$  (of  $\mathcal{W}_F$  or  $\mathrm{GL}_n(F)$ ) by tame characters of a tamely ramified field extension of  $F$ , canonically associated to  $\pi$ . The authors show this operation is preserved by the Langlands correspondence.

The Hamiltonian  $\int_X (|\nabla u|^2 + m^2 u^2) dx$ , defined on functions on  $R \times X$ , where  $X$  is a compact manifold, has critical points which are solutions of the linear Klein-Gordon equation. The author considers perturbations of this Hamiltonian, given by polynomial expressions depending on first order derivatives of  $u$ . The associated PDE is then a quasi-linear Klein-Gordon equation. The author shows that, when  $X$  is the sphere, and when the mass parameter  $m$  is outside an exceptional subset of zero measure, smooth Cauchy data of small size  $\epsilon$  give rise to almost global solutions, i.e. solutions defined on a time interval of length  $cN\epsilon$  for any  $N$ . Previous results were limited either to the semi-linear case (when the perturbation of the Hamiltonian depends only on  $u$ ) or to the one dimensional problem. The proof is based on a quasi-linear version of the Birkhoff normal forms method, relying on convenient generalizations of para-differential calculus.

Consider a Hamiltonian action of a compact connected Lie group on a symplectic manifold  $M$ . Conjecturally, under suitable assumptions there exists a morphism of cohomological field theories from the equivariant Gromov-Witten theory of  $M$  to the Gromov-Witten theory of the symplectic quotient. The morphism should be a deformation of the Kirwan map. The idea, due to D. A. Salamon, is to define such a deformation by counting gauge equivalence classes of symplectic vortices over the complex plane  $\mathbb{C}$ . The present memoir is part of a project whose goal is to make this definition rigorous. Its main results deal with the symplectically aspherical case.

The goal of this work is to propose a finite population counterpart to Eigen's model, which incorporates stochastic effects. The author considers a Moran model describing the evolution of a population of size  $N$  of chromosomes of length  $L$  over an alphabet of cardinality  $\Sigma$ . The mutation probability per locus is  $\mu$ . He deals only with the sharp peak landscape: the replication rate is  $r_0$  for the master sequence and  $r_1$  for the other sequences. He studies the equilibrium distribution of the process in the regime where

A stationary solution of the rotating Navier-Stokes equations with a boundary condition is called an Ekman boundary layer. This book constructs stationary solutions of the rotating Navier-Stokes-Boussinesq equations with stratification effects in the case when the rotating axis is not necessarily perpendicular to the horizon. The author calls such stationary solutions Ekman layers. This book shows the existence of a weak solution to an Ekman perturbed system, which satisfies the strong energy inequality. Moreover, the author discusses the uniqueness of weak solutions and computes the decay rate of weak solutions with respect to time under some assumptions on the Ekman layers and the physical parameters. The author also shows that there exists a unique global-in-time strong solution of the perturbed system when the initial datum is sufficiently small. Comparing a weak solution satisfying the strong energy inequality with the strong solution implies that the weak solution is smooth with respect to time when time is sufficiently large.

The authors develop a theory for the existence of perfect matchings in hypergraphs under quite general conditions. Informally speaking, the obstructions to perfect matchings are geometric, and are of two distinct types: 'space barriers' from convex geometry, and 'divisibility barriers' from arithmetic lattice-based constructions. To formulate precise results, they introduce the setting of simplicial complexes with minimum degree sequences, which is a generalisation of the usual minimum degree condition. They determine the essentially best possible minimum degree sequence for finding an almost perfect matching. Furthermore, their main result establishes the stability property: under the same degree assumption, if there is no perfect matching then there must be a space or divisibility barrier. This allows the use of the stability method in proving exact results. Besides recovering previous results, the authors apply our theory to the solution of two open problems on hypergraph packings: the minimum degree threshold for packing tetrahedra in  $k$ -graphs, and Fischer's conjecture on a multipartite form of the Hajnal-Szemerédi Theorem. Here they prove the exact result for tetrahedra and the asymptotic result for Fischer's conjecture; since the exact result for the latter is technical they defer it to a subsequent paper.

The authors consider the time-dependent Schrödinger equation on a Riemannian manifold with a potential that localizes a certain subspace of states close to a fixed submanifold  $S$ . When the authors scale the potential in the directions normal to  $S$  by a parameter  $\epsilon$ , the solutions concentrate in an  $\epsilon$ -neighborhood of  $S$ . This situation occurs for example in quantum wave guides and for the motion of nuclei in electronic potential surfaces in quantum molecular dynamics. The authors derive an effective Schrödinger equation on the submanifold and show that its solutions, suitably lifted to  $M$ , approximate the solutions of the original equation on up to errors of order  $\epsilon^2$  at time  $t$ . Furthermore, the authors prove that the eigenvalues of

the corresponding effective Hamiltonian below a certain energy coincide up to errors of order with those of the full Hamiltonian under reasonable conditions.

In this monograph the authors introduce a new method to study bifurcations of KAM tori with fixed Diophantine frequency in parameter-dependent Hamiltonian systems. It is based on Singularity Theory of critical points of a real-valued function which the authors call the potential. The potential is constructed in such a way that: nondegenerate critical points of the potential correspond to twist invariant tori (i.e. with nondegenerate torsion) and degenerate critical points of the potential correspond to non-twist invariant tori. Hence, bifurcating points correspond to non-twist tori.

The classical Grothendieck inequality is viewed as a statement about representations of functions of two variables over discrete domains by integrals of two-fold products of functions of one variable. An analogous statement is proved, concerning continuous functions of two variables over general topological domains. The main result is the construction of a continuous map  $\Phi$  from  $L^2(A)$  into  $L^2(\Omega_A, \mathbb{P}_A)$ , where  $A$  is a set,  $\Omega_A = \{-1, 1\}^A$ , and  $\mathbb{P}_A$  is the uniform probability measure on  $\Omega_A$ .

The authors investigate the global continuity on spaces with of Fourier integral operators with smooth and rough amplitudes and/or phase functions subject to certain necessary non-degeneracy conditions. In this context they prove the optimal global boundedness result for Fourier integral operators with non-degenerate phase functions and the most general smooth Hörmander class amplitudes i.e. those in with . They also prove the very first results concerning the continuity of smooth and rough Fourier integral operators on weighted spaces, with and (i.e. the Muckenhoupt weights) for operators with rough and smooth amplitudes and phase functions satisfying a suitable rank condition.

The authors develop elements of a general dilation theory for operator-valued measures. Hilbert space operator-valued measures are closely related to bounded linear maps on abelian von Neumann algebras, and some of their results include new dilation results for bounded linear maps that are not necessarily completely bounded, and from domain algebras that are not necessarily abelian. In the non-cb case the dilation space often needs to be a Banach space. They give applications to both the discrete and the continuous frame theory. There are natural associations between the theory of frames (including continuous frames and framings), the theory of operator-valued measures on sigma-algebras of sets, and the theory of continuous linear maps between -algebras. In this connection frame theory itself is identified with the special case in which the domain algebra for the maps is an abelian von Neumann algebra and the map is normal (i.e. ultraweakly, or weakly, or  $w^*$ ) continuous.

Polynomial approximation on convex polytopes in is considered in uniform and -norms. For an appropriate modulus of smoothness matching direct and converse estimates are proven. In the -case so called strong direct and converse results are also verified. The equivalence of the moduli of smoothness with an appropriate -functional follows as a consequence. The results solve a problem that was left open since the mid 1980s when some of the present findings were established for special, so-called simple polytopes.

Introduction Statement of the results Mixing time preliminaries Outline of the proof of Theorem 2.1 Random graph estimates Supercritical case Subcritical case Critical Case Fast mixing of the Swendsen-Wang process on trees Acknowledgements Bibliography

- Solved Board Examination Paper 2020 along with CBSE Marking Scheme from 2016 to 2019 for in-depth study. •
- Previous Years' Board Examination Questions with Solutions from March 2016 to March 2019 to facilitate focused study. •
- Handwritten Toppers' Answer sheets from 2016-2019 for perfection in answering Board Examination Questions •

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It has, improbably, been called uncommonly lucid, even riveting by The New York Times, and it was a finalist for the 2004 National Book Awards nonfiction honor. It is a literally chilling read, especially in its minute-by-minute description of the events of the morning of 9/11 inside the Twin Towers. It is The 9/11 Commission Report, which was, before its publication, perhaps one of the most anticipated government reports of all time, and has been since an unlikely bestseller. The official statement by the National Commission on Terrorist Attacks Upon the United States—which was instituted in late 2002 and chaired by former New Jersey Governor Thomas Kean—it details what went wrong on that day (such as intelligence failures), what went right (the heroic response of emergency services and self-organizing civilians), and how to avert similar future attacks. Highlighting evidence from the day, from airport surveillance footage of the terrorists to phone calls from the doomed flights, and offering details that have otherwise gone unheard, this is an astonishing firsthand document of contemporary history. While controversial in parts—it has been criticized for failing to include testimony from key individuals, and it completely omits any mention of the mysterious collapse of WTC 7—it is nevertheless an essential record of one of the most transformational events of modern times.

The authors consider the Schrödinger Map equation in  $2+1$  dimensions, with values into  $\mathbb{S}^2$ . This admits a lowest energy steady state  $Q$ , namely the stereographic projection, which extends to a two dimensional family of steady states by scaling and rotation. The authors prove that  $Q$  is unstable in the energy space  $\dot{H}^1$ . However, in the process of proving this they also show that within the equivariant class  $Q$  is stable in a stronger topology  $X \subset \dot{H}^1$ .

For a finite real reflection group  $W$  and a  $W$ -orbit  $\mathcal{O}$  of flats in its reflection arrangement—or equivalently a conjugacy class of its parabolic subgroups—the authors introduce a statistic  $\operatorname{noninv}_{\mathcal{O}}(w)$  on  $W$  in  $W$  that counts the number of “ $\mathcal{O}$ -noninversions” of  $w$ . This generalizes the classical (non-)inversion statistic for permutations  $w$  in the symmetric group  $\mathfrak{S}_n$ . The authors then study the operator  $\nu_{\mathcal{O}}$  of right-multiplication within the group algebra  $\mathbb{C}W$  by the element that has  $\operatorname{noninv}_{\mathcal{O}}(w)$  as its coefficient on  $w$ .

Spectral triples for nonunital algebras model locally compact spaces in noncommutative geometry. In the present text,

the authors prove the local index formula for spectral triples over nonunital algebras, without the assumption of local units in our algebra. This formula has been successfully used to calculate index pairings in numerous noncommutative examples. The absence of any other effective method of investigating index problems in geometries that are genuinely noncommutative, particularly in the nonunital situation, was a primary motivation for this study and the authors illustrate this point with two examples in the text. In order to understand what is new in their approach in the commutative setting the authors prove an analogue of the Gromov-Lawson relative index formula (for Dirac type operators) for even dimensional manifolds with bounded geometry, without invoking compact supports. For odd dimensional manifolds their index formula appears to be completely new.

A partial solution of the quaternionic contact Yamabe problem on the quaternionic sphere is given. It is shown that the torsion of the Biquard connection vanishes exactly when the trace-free part of the horizontal Ricci tensor of the Biquard connection is zero and this occurs precisely on 3-Sasakian manifolds. All conformal transformations sending the standard flat torsion-free quaternionic contact structure on the quaternionic Heisenberg group to a quaternionic contact structure with vanishing torsion of the Biquard connection are explicitly described. A "3-Hamiltonian form" of infinitesimal conformal automorphisms of quaternionic contact structures is presented.

The author develops a homology theory for Smale spaces, which include the basics sets for an Axiom A diffeomorphism. It is based on two ingredients. The first is an improved version of Bowen's result that every such system is the image of a shift of finite type under a finite-to-one factor map. The second is Krieger's dimension group invariant for shifts of finite type. He proves a Lefschetz formula which relates the number of periodic points of the system for a given period to trace data from the action of the dynamics on the homology groups. The existence of such a theory was proposed by Bowen in the 1970s.

The authors study the complex geometry and coherent cohomology of nonclassical Mumford-Tate domains and their quotients by discrete groups. Their focus throughout is on the domains which occur as open  $\mathbb{C}^*$ -orbits in the flag varieties for  $SO^*(2n+1)$  and  $U^*(n)$ , regarded as classifying spaces for Hodge structures of weight three. In the context provided by these basic examples, the authors formulate and illustrate the general method by which correspondence spaces give rise to Penrose transforms between the cohomologies of distinct such orbits with coefficients in homogeneous line bundles.

Descriptive set theory is mainly concerned with studying subsets of the space of all countable binary sequences. In this paper the authors study the generalization where countable is replaced by uncountable. They explore properties of generalized Baire and Cantor spaces, equivalence relations and their Borel reducibility. The study shows that the descriptive set theory looks very different in this generalized setting compared to the classical, countable case. They also draw the connection between the stability theoretic complexity of first-order theories and the descriptive set theoretic complexity of their isomorphism relations. The authors' results suggest that Borel reducibility on uncountable structures is a model theoretically natural way to compare the complexity of isomorphism relations.

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