Basic Commutative Algebra

This book can be understood as a model for teaching commutative algebra, and takes into account modern developments such as algorithmic and computational aspects. As soon as a new concept is introduced, the authors show how the concept can be worked on using a computer. The computations are exemplified with the computer algebra system Singular, developed by the authors. Singular is a special system for polynomial computation with many features for global as well as for local commutative algebra and algebraic geometry. The book includes a CD containing Singular as well as the examples and procedures explained in the book.

Translated from the popular French edition, this book offers a detailed introduction to various basic concepts, methods, principles, and results of commutative algebra. It takes a constructive viewpoint in commutative algebra and studies algorithmic approaches alongside several abstract classical theories. Indeed, it revisits these traditional topics with a new and simplifying manner, making the subject both accessible and innovative. The algorithmic aspects of such naturally abstract topics as Galois theory, Dedekind rings, Prüfer rings, finitely generated projective modules, dimension theory of commutative rings, and others in the

current treatise, are all analysed in the spirit of the great developers of constructive algebra in the nineteenth century. This updated and revised edition contains over 350 well-arranged exercises, together with their helpful hints for solution. A basic knowledge of linear algebra, group theory, elementary number theory as well as the fundamentals of ring and module theory is required. Commutative Algebra: Constructive Methods will be useful for graduate students, and also researchers, instructors and theoretical computer scientists.
This introduction to polynomial rings, Gröbner bases and applications bridges the gap in the literature between theory and actual computation. It details numerous applications, covering fields as disparate as algebraic geometry and financial markets. To aid in a full understanding of these applications, more than 40 tutorials illustrate how the theory can be used. The book also includes many exercises, both theoretical and practical.
The constructive approach to mathematics has enjoyed a renaissance, caused in large part by the appearance of Errett Bishop's book Foundations of constr"uctiue analysis in 1967, and by the subtle influences of the proliferation of powerful computers. Bishop demonstrated that pure mathematics can be developed from a constructive point of view while maintaining a continuity with classical terminology and spirit; much more of classical mathematics was

preserved than had been thought possible, and no classically false theorems resulted, as had been the case in other constructive schools such as intuitionism and Russian constructivism. The computers created a widespread awareness of the intuitive notion of an effecti ve procedure, and of computation in principle, in addi tion to stimulating the study of constructive algebra for actual implementation, and from the point of view of recursive function theory. In analysis, constructive problems arise instantly because we must start with the real numbers, and there is no finite procedure for deciding whether two given real numbers are equal or not (the real numbers are not discrete) . The main thrust of constructive mathematics was in the direction of analysis, although several mathematicians, including Kronecker and van der waerden, made important contributions to construc tive algebra. Heyting, working in intuitionistic algebra, concentrated on issues raised by considering algebraic structures over the real numbers, and so developed a handmaiden'of analysis rather than a theory of discrete algebraic structures.

The author is well known to the math community as an excellent researcher and expositor in the fields of combinatorics and algebra. His first book was widely read. In this second edition, which contains a significant amount of new material in areas of current

interest, Stanley, as before, presents the "big picture" in an engaging framework.
Algebraic geometry is a fascinating branch of mathematics that combines methods from both, algebra and geometry. It transcends the limited scope of pure algebra by means of geometric construction principles. Moreover, Grothendieck's schemes invented in the late 1950s allowed the application of algebraic-geometric methods in fields that formerly seemed to be far away from geometry, like algebraic number theory. The new techniques paved the way to spectacular progress such as the proof of Fermat's Last Theorem by Wiles and Taylor. The scheme-theoretic approach to algebraic geometry is explained for non-experts. More advanced readers can use the book to broaden their view on the subject. A separate part deals with the necessary prerequisites from commutative algebra. On a whole, the book provides a very accessible and self-contained introduction to algebraic geometry, up to a quite advanced level. Every chapter of the book is preceded by a motivating introduction with an informal discussion of the contents. Typical examples and an abundance of exercises illustrate each section. This way the book is an excellent solution for learning by yourself or for complementing knowledge that is already present. It can equally be used as a convenient source for courses and seminars or as supplemental literature.

Commutative algebra is a rapidly growing subject that is developing in many different directions. This volume presents several of the most recent results from various areas related to both Noetherian and non-Noetherian commutative algebra. This volume contains a collection of invited survey articles by some of the leading experts in the field. The authors of these chapters have been carefully selected for their important contributions to an area of commutative-algebraic research. Some topics presented in the volume include: generalizations of cyclic modules, zero divisor graphs, class semigroups, forcing algebras, syzygy bundles, tight closure, Gorenstein dimensions, tensor products of algebras over fields, as well as many others. This book is intended for researchers and graduate students interested in studying the many topics related to commutative algebra.

This ACM volume deals with tackling problems that can be represented by data structures which are essentially matrices with polynomial entries, mediated by the disciplines of commutative algebra and algebraic geometry. The discoveries stem from an interdisciplinary branch of research which has been growing steadily over the past decade. The author covers a wide range, from showing how to obtain deep heuristics in a computation of a ring, a module or a morphism, to developing means of solving nonlinear systems of equations - highlighting the use of advanced techniques to bring down the cost of computation. Although intended for advanced students and researchers with interests both in algebra and computation, many parts may be read by anyone with a basic abstract algebra course.

From the Preface: "topics are: (a) valuation theory; (b) theory of polynomial and power series rings (including generalizations to graded rings and modules); (c) local algebra... the algebro-geometric connections and applications of the purely algebraic material are constantly stressed and abundantly scattered throughout the exposition. Thus, this volume can be used in part as an introduction to some basic concepts and the arithmetic foundations of algebraic geometry."

This textbook offers a thorough, modern introduction into commutative algebra. It is intented mainly to serve as a guide for a course of one or two semesters, or for self-study. The carefully selected subject matter concentrates on the concepts and results at the center of the field. The book maintains a constant view on the natural geometric context, enabling the reader to gain a deeper understanding of the material. Although it emphasizes theory, three chapters are devoted to computational aspects. Many illustrative examples and exercises enrich the text.

From the reviews: "The book is well written. We find here many examples. Each chapter is followed by exercises, and at the end of the book there are outline solutions to some of them. [...] I especially appreciated the lively style of the book; [...] one is quickly able to find necessary details." EMS Newsletter

An introduction to abstract algebraic geometry, with the only prerequisites being results from commutative algebra, which are stated as needed, and some elementary topology. More than 400 exercises distributed throughout the book offer specific examples as well as more specialised topics not treated in the main text, while three appendices present brief accounts of some areas of current research. This book can thus be used as textbook for an introductory course in algebraic geometry following a basic graduate course in algebra. Robin Hartshorne studied algebraic geometry with Oscar Zariski and David Mumford at Harvard, and with J.-P. Serre and A. Grothendieck in Paris. He is the author of "Residues and Duality", "Foundations of Projective Geometry", "Ample Subvarieties of Algebraic Varieties", and numerous research titles.

Designed for a one-semester course in mathematics, this textbook presents a concise and practical introduction to commutative algebra in terms of normal (normalized) structure. It shows how the nature of commutative algebra has been used by both number theory and algebraic geometry. Many worked examples and a number of problem (with hints) can be found in the volume. It is also a convenient reference for researchers who use basic commutative algebra.

Originally published in 1985, this classic textbook is an English translation of Einführung in die kommutative Algebra und algebraische Geometrie. As part of the Modern Birkhäuser Classics series, the publisher is proud to make Introduction to Commutative Algebra and Algebraic Geometry available to a wider audience.

Aimed at students who have taken a basic course in algebra, the goal of the text is to present important results concerning the representation of algebraic varieties as intersections of the least possible number of hypersurfaces and—a closely related problem—with the most economical generation of ideals in Noetherian rings. Along the way, one encounters many basic concepts of commutative algebra and algebraic geometry and proves many facts which can then serve as a basic stock for a deeper study of these subjects. Recent developments are covered Contains over 100 figures and 250 exercises Includes complete proofs An Introduction to Commutative Algebra and Number Theory is an elementary introduction to these subjects. Beginning with a concise review of groups, rings and fields, the author presents topics in algebra from a distinctly number-theoretic perspective and sprinkles number theory results throughout his presentation. The topics in algebra include polynomial rings, UFD, PID, and Euclidean domains; and field extensions, modules, and Dedekind domains. In the section on number theory, in addition to covering elementary congruence results, the laws of quadratic reciprocity and basics of algebraic number fields, this book gives glimpses into some deeper aspects of the subject. These include Warning's and Chevally's theorems in the finite field sections, and many results of additive number theory, such as the derivation of LaGrange's four-square theorem from Minkowski's result in the geometry of numbers. With addition of remarks and comments and with references in the bibliography, the author stimulates readers to

explore the subject beyond the scope of this book.
This book provides a concise yet comprehensive and self-contained introduction to Grobner basis theory and its applications to various current research topics in commutative algebra. It especially aims to help young researchers become acquainted with fundamental tools and techniques related to Grobner bases which are used in commutative algebra and to arouse their interest in exploring further topics such as toric rings, Koszul and Rees algebras, determinantal ideal theory, binomial edge ideals, and their applications to statistics. The book can be used for graduate courses and self-study. More than 100 problems will help the readers to better understand the main theoretical results and will inspire them to further investigate the topics studied in this book.
This book provides careful and detailed introductions to some of the latest advances in three significant areas of rapid development in commutative algebra and its applications. The book is based on courses at the Winter School on Commutative Algebra and Applications held in Barcelona: Tight closure and vector bundles, by H. Brenner; Combinatorics and commutative algebra, by J. Herzog; and Constructive desingularization, by O. Villamayor. The exposition is aimed at graduate students who have some experience with basic commutative algebra or algebraic geometry but may also serve as an introduction to these modern approaches for mathematicians already familiar with commutative algebra. This book is published in cooperation with Real Sociedad Matematica Espanola.
This second volume of our treatise on commutative algebra deals largely with three basic topics, which go beyond the more or less classical material of volume I and are on the whole of a more advanced nature and a more recent vintage. These topics are: (a) valuation theory; (b) theory of

polynomial and power series rings (including generalizations to graded rings and modules); (c) local algebra. Because most of these topics have either their source or their best motivation in algebraic geom etry, the algebro-geometric connections and applications of the purely algebraic material are constantly stressed and abundantly scattered through out the exposition. Thus, this volume can be used in part as an introduc tion to some basic concepts and the arithmetic foundations of algebraic geometry. The reader who is not immediately concerned with geometric applications may omit the algebro-geometric material in a first reading (see" Instructions to the reader," page vii), but it is only fair to say that many a reader will find it more instructive to find out immediately what is the geometric motivation behind the purely algebraic material of this volume. The first 8 sections of Chapter VI (including § 5bis) deal directly with properties of places, rather than with those of the valuation associated with a place. These, therefore, are properties of valuations in which the value group of the valuation is not involved.
A precise, fundamental study of commutative algebra, this largely self-contained treatment is the first in a two-volume set. Intended for advanced undergraduates and graduate students in mathematics, its prerequisites are the rudiments of set theory and linear algebra, including matrices and determinants. The opening chapter develops introductory notions concerning groups, rings, fields, polynomial rings, and vector spaces. Subsequent chapters feature an exposition of field theory and classical material concerning ideals and modules in arbitrary commutative rings, including detailed studies of direct sum decompositions. The final two chapters explore Noetherian rings and Dedekind domains. This work prepares readers for the more advanced topics of Volume II, which include valuation theory, polynomial and power series rings, and local algebra.

This book stems from lectures on commutative algebra for 4th-year university students at two French universities (Paris and Rennes). At that level, students have already followed a basic course in linear algebra and are essentially fluent with the language of vector spaces over fields. The topics introduced include arithmetic of rings, modules, especially principal ideal rings and the classification of modules over such rings, Galois theory, as well as an introduction to more advanced topics such as homological algebra, tensor products, and algebraic concepts involved in algebraic geometry. More than 300 exercises will allow the reader to deepen his understanding of the subject. The book also includes 11 historical vignettes about mathematicians who contributed to commutative algebra.

The central theme of this volume is commutative algebra, with emphasis on special graded algebras, which are increasingly of interest in problems of algebraic geometry, combinatorics and computer algebra. Most of the papers have partly survey character, but are research-oriented, aiming at classification and structural results.

The purpose of this book is twofold: to present some basic ideas in commutative algebra and algebraic geometry and to introduce topics of current research, centered around the themes of Gröbner bases, resultants and syzygies. The presentation of the material combines definitions and proofs with an emphasis on concrete examples. The authors illustrate the use of software such as Mathematica and Singular. The design of the text in each chapter consists of two parts: the fundamentals and the applications, which make it suitable for courses of various lengths, levels, and topics based on the mathematical background of the students. The fundamentals portion of the chapter is intended to be read with minimal outside assistance, and to learn some of the most useful tools in commutative algebra. The applications of the chapter are to provide a glimpse of the advanced mathematical research where the topics and results are related to the material presented earlier. In the applications portion, the authors present a number of results from a wide range of sources without detailed proofs. The applications portion of the chapter is suitable for a reader who knows a little commutative algebra and algebraic geometry already, and serves as a guide to some interesting research topics. This book should be thought of as an introduction to more advanced texts and research topics. Its novelty is that the material presented is a unique combination of the essential methods and the current research results. The goal is to equip readers with the fundamental classical algebra and geometry tools, ignite their research interests, and initiate some potential research projects in the related areas.

There is no shortage of books on Commutative Algebra, but the present book is different. Most books are monographs, with extensive coverage. There is one notable exception: Atiyah and Macdonald’s 1969 classic. It is a clear, concise, and efficient textbook, aimed at beginners, with a good selection of topics. So it has remained popular. However, its age and flaws do show. So there is need for an updated and improved version, which the present book aims to be.

Relations between groups and sets, results and methods of abstract algebra in terms of number theory and geometry, and noncommutative and homological algebra. Solutions. 2006 edition.

For those looking for an introduction to the area of commutative algebra, this book opens all the right doors and provides a clarity of understanding that all will welcome.

The main goal of this book is to find the constructive content hidden in abstract proofs of concrete theorems in Commutative Algebra, especially in well-known theorems concerning projective modules over polynomial rings (mainly the Quillen-Suslin theorem) and syzygies of multivariate polynomials with coefficients in a valuation ring. Simple and constructive proofs of some results in the theory of projective modules over polynomial rings are also given, and light is cast upon recent progress on the Hermite ring and Gröbner ring conjectures. New conjectures on unimodular completion arising from our constructive approach to the unimodular completion problem are presented. Constructive algebra can be understood as a first preprocessing step for computer algebra that leads to the discovery of general algorithms, even if they are sometimes not efficient. From a logical point of view, the dynamical evaluation gives a constructive substitute for two highly nonconstructive tools of abstract algebra: the Law of Excluded Middle and Zorn's Lemma. For instance, these tools are required in order to construct the complete prime factorization of an ideal in a Dedekind ring, whereas the dynamical method reveals the computational content of this construction. These lecture notes follow this dynamical philosophy.

This contributed volume brings together the highest quality expository papers written by leaders and talented junior mathematicians in the field of Commutative Algebra. Contributions cover a very wide range of topics, including core areas in Commutative Algebra and also relations to Algebraic Geometry, Algebraic Combinatorics, Hyperplane Arrangements, Homological Algebra, and String Theory. The book aims to showcase the area, especially for the benefit of junior mathematicians and researchers who are new to the field; it will aid them in broadening their background and to gain a deeper understanding of the current research in this area. Exciting developments are surveyed and many open problems are discussed with the aspiration to inspire the readers and foster further research.

This book explores commutative ring theory, an important a foundation for algebraic geometry and complex analytical geometry.

Algebraic Geometry and Commutative Algebra in Honor of Masayoshi Nagata presents a collection of papers on algebraic geometry and commutative algebra in honor of Masayoshi Nagata for his significant contributions to commutative algebra. Topics covered range from power series rings and rings of invariants of finite linear groups to the convolution algebra of distributions on totally disconnected locally compact groups. The discussion begins with a description of several formulas for enumerating certain types of objects, which may be tabular arrangements of integers called Young tableaux or some types of monomials. The next chapter explains how to establish these enumerative formulas, with emphasis on the role played by transformations of determinantal polynomials and recurrence relations satisfied by them. The book then turns to several applications of the enumerative formulas and universal identity, including including enumerative proofs of the straightening law of Doubilet-Rota-Stein and computations of Hilbert functions of polynomial ideals of certain determinantal loci. Invariant differentials and quaternion extensions are also examined, along with the moduli of Todorov surfaces and the classification problem of embedded lines in characteristic p. This monograph will be a useful resource for practitioners and researchers in algebra and geometry.

This book is an expanded text for a graduate course in commutative algebra, focusing on the algebraic underpinnings of algebraic geometry and of number theory. Accordingly, the theory of affine algebras is featured, treated both directly and via the theory of Noetherian and Artinian modules, and the theory of graded algebras is included to provide the foundation for projective varieties. Major topics include the theory of modules over a principal ideal domain,

and its applicationsto matrix theory (including the Jordan decomposition), the Galois theory of field extensions, transcendence degree, the prime spectrum of an algebra, localization, and the classical theory of Noetherian and Artinian rings. Later chapters include some algebraic theory of elliptic curves (featuring theMordell-Weil theorem) and valuation theory, including local fields. One feature of the book is an extension of the text through a series of appendices. This permits the inclusion of more advanced material, such as transcendental field extensions, the discriminant and resultant, the theory of Dedekind domains, and basic theorems of rings of algebraic integers. An extended appendix on derivations includes the Jacobian conjecture and Makar-Limanov's theory of locally nilpotent derivations. Grobnerbases can be found in another appendix. Exercises provide a further extension of the text. The book can be used both as a textbook and as a reference source.
This is a comprehensive review of commutative algebra, from localization and primary decomposition through dimension theory, homological methods, free resolutions and duality, emphasizing the origins of the ideas and their connections with other parts of mathematics. The book gives a concise treatment of Grobner basis theory and the constructive methods in commutative algebra and algebraic geometry that flow from it. Many exercises included.

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This book is the first to be devoted entirely to fuzzy abstract algebra. It presents an up-to-date version of fuzzy commutative algebra, and focuses on the connection between L-subgroups of a group, and L-subfields of a field. In particular, an up-to-date treatment of nonlinear systems of fuzzy intersection equations is given. Contents:L-Subsets and L-SubgroupsL-Subgroups of Abelian GroupsL-Subrings and L-IdealsL-SubmodulesL-SubfieldsStructure of L-Subrings and L-IdealsAlgebraic L-Varieties and Intersection EquationsL-SubspacesGalois Theory and Group L-Subalgebras Readership: Pure mathematicians. Keywords:Fuzzy Subsets;Fuzzy Subgroups;Fuzzy Subrings;Fuzzy Ideals;Fuzzy Submodules ;Fuzzy Subfields;Fuzzy Varieties;Fuzzy Subspaces;Fuzzy Galois Theory;Fuzzy Group SubalgebrasReviews: “The book is self-contained … This will serve as a nice reference book for researchers in the field. It may also be used as a good text for an advanced graduate course. There are a good number of exercises at the end of each chapter of the book.” Mathematical Reviews

This introduction to commutative algebra gives an account of some general properties of rings and modules, with their applications to number theory and geometry. It assumes only that the reader has completed an undergraduate algebra course. The fresh approach and simplicity of proof enable a large amount of material to be covered; exercises and examples are included throughout the notes.

This textbook, set for a one or two semester course in commutative algebra, provides an introduction to commutative algebra at the postgraduate and research levels. The main prerequisites are familiarity with groups, rings and fields. Proofs are self-contained. The book will be useful to beginners and experienced researchers alike. The material is so arranged that the beginner can learn through self-study or by attending a course. For the experienced researcher, the book may serve to present new perspectives on some well-known results, or as a reference.

Abstract: "While computer algebra systems have dealt with polynomials and rational functions with integer coefficients for many years, dealing with more general constructs from commutative algebra is a more recent problem. In this paper we explain how one system solves this problem, what types and operators it is necessary to introduce and, in short, how one can construct a computational theory of commutative algebra. Of necessity, such a theory is rather different from the conventional, non-constructive, theory. It is also somewhat different from the theories of Seidenberg (1974) and his school, who are not particularly concerned with practical questions of efficiency."

Introductory account of commutative algebra, aimed at students with a background in basic algebra.

Contains contributions by over 25 leading international mathematicians in the areas of commutative algebra and algebraic geometry. The text presents developments and results based on, and inspired by, the work of Mario Fiorentini. It covers topics ranging from almost numerical invariants of algebraic curves to deformation of projective schemes.